Tuning of Finite Element Model Parameters to Match Nonlinear Reduced Order Models

Kyusic Park[1] and Matthew S. Allen[2]

[1]Graduate Student; UW-Madison - Department of Mechanical Engineering,
[2] Professor, UW-Madison - Department of Engineering Physics
email: kpark93@wisc.edu matt.allen@wisc.edu

Abstract

There has been a growing demand for nonlinear model updating procedures in the structural dynamics community as advanced vehicles have started to incorporate nonlinearity into their designs. Finite element (FE) model updating is difficult for a nonlinear structure for several reasons: there may be many unknown parameters in the FE model so that multiple solutions may exist, and it is very expensive to compute the structure's nonlinear normal modes from a full FE model that are an excellent metric to use for nonlinear updating. A recent work showed that some of these challenges can be overcome by updating a nonlinear reduced order model (ROM) rather than the full FE model. The updated ROM can be used to compute response statistics, stresses and ultimately life of the structure. However, one drawback of this approach is that one does not gain physical insight into which parameters in the FE model were in error, and so it is difficult to transfer the lessons learned to future FE models. This work explores the feasibility of updating a FE model to correlate with a ROM with a set of known parameters. The target ROM parameters consist of the linear stiffness and the nonlinear stiffness coefficients which have been identified previously by updating the ROM to correlate with experimental measurements. In this process an optimization routine is setup in which the free parameters are those of the FE model, such as boundary stiffness springs, imperfections, pre-stress, etc. The optimization procedure is wrapped around a ROM creation algorithm that iteratively tunes the FE parameters, creates a ROM, and evaluates the ROM parameters with respect to the target ROM in order to minimize the objective function. This work will demonstrate the procedure on a numerical example of a curved beam in order to verify its effectiveness on the nonlinear model updating.

Keywords: Nonlinear Dynamics, Nonlinear Model Updating, Reduced Order Models, Nonlinear Normal Modes, Gradient-based Optimization

1 Introduction

This work demonstrates a new approach to model updating in which a finite element (FE) model is updated to correlate with a reduced order model (ROM), which is already known to correlate well with measurements. The framework optimizes the FE model parameters by a gradient based method so that the ROM of the FE model matches a target ROM, which has presumably been derived experimentally [1]. The following section briefly outlines the reduced order modeling and an overview of the FE parameter tuning framework. Then, a numerical example of a curved beam is presented and discussed.

2 Theory

The n DOF FE model of a geometrically nonlinear structure can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{f}_{\text{nl}}(\mathbf{x}) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the $n \times n$ mass, damping, and linear stiffness matrices respectively. The $n \times 1$ nonlinear restoring force $\mathbf{f}_{\text{nl}}$ is a function of the displacement vector $\mathbf{x}$. The full equations of motion of the FE model in a physical space can be projected onto a modal subspace by using $\mathbf{x}(t) = \mathbf{\Phi}_m\mathbf{q}(t)$, where $\mathbf{\Phi}_m$ is the n x m mass normalized mode shape matrix and $\mathbf{q}$ is the m x 1 modal displacement. The reduced DOF of modal coordinates can be significantly smaller than the DOF of original physical coordinates ($m \ll n$). Then, the r^{th} nonlinear modal equation becomes

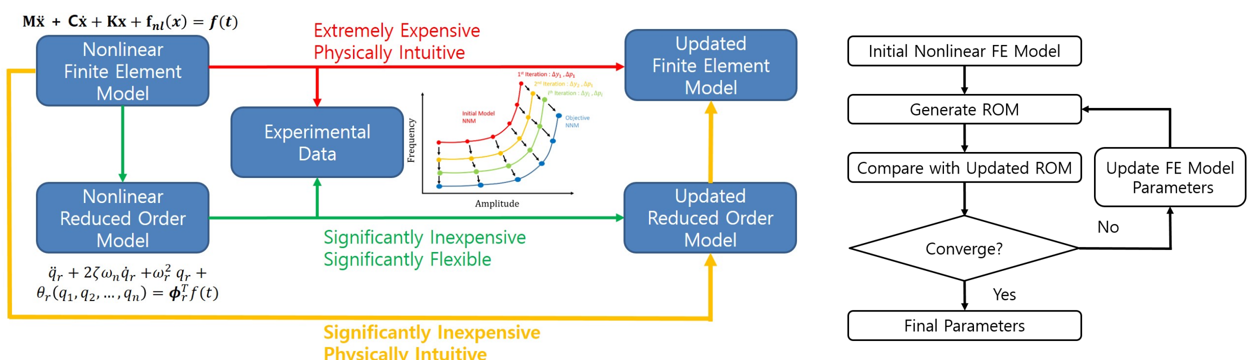
$$\ddot{q}_r + c_r\dot{q}_r + \omega_r^2 q_r + \theta_r(q_1, q_2, ..., q_m) = \mathbf{\Phi}_r^T \mathbf{f}(t) \tag{2}$$

where the nonlinear restoring force θ_r is a function of the modal displacements as $\theta_r(\mathbf{q}) = \mathbf{\Phi}_r^T \mathbf{f}_{\text{nl}}(\mathbf{\Phi}_m \mathbf{q})$. The nonlinear restoring force that captures the geometric nonlinearity of a linear elastic system can be well approximated as

$$\theta_r(q_1, q_2, ...q_m) = \sum_{i=1}^{m}\sum_{j=i}^{m} A_r(i,j)q_i q_j + \sum_{i=1}^{m}\sum_{j=i}^{m}\sum_{k=j}^{m} B_r(i,j,k)q_i q_j q_k \tag{3}$$

where A_r and B_r are the quadratic and cubic nonlinear stiffness terms, respectively. This work uses the implicit condensation and expansion (ICE) method in order to find the nonlinear stiffness terms, which was presented in detail in [2].

The proposed procedure is illustrated in Figure 1a, which follows the yellow path of the flowchart. It uses an original FE model and a target ROM that was previously updated toward an experimental system. In fact, instead of directly updating the FE model parameters to those of an actual system (red path in the flowchart), their mismatches were incorporated into a ROM so that its resulting nonlinear stiffness terms could be updated toward the experimental data at a significantly reduced computational cost (green path in the flowchart). Then, our framework updates the original FE model to match with the updated ROM by tuning a set of design variables that can be chosen from the FE model parameters, such as imperfections in geometry, pre-stress distribution, boundary conditions for stiffness, etc. The optimization involves a loop in which the FE model parameters are tuned, a ROM is created by the ICE method, and the ROM coefficients are evaluated with respect to the target ROM coefficients until their discrepancies are minimized (Figure 1b). The optimization routine finds the minimum objective function based on a gradient based method, i.e. the interior-point algorithm [3], which is implemented in the MATLAB® function, *fmincon*.



(a)

(b)

Figure 1: Representation of the finite element model tuning framework: (a) an overall flow diagram and (b) an optimization diagram

3 Numerical Example - Curved beam

In this work, the proposed procedure is tested numerically on the FE model of the curved beam with axial boundary springs in Figure 2. The beam had length of 9 in., width of 0.5 in., thickness of 0.031 in., and radius of curvature of 80 in.. The Young"s modulus was 29,700 ksi, Poisson's ratio was 0.28 and mass density was 7.36 $\times 10^4$ lb-s^2/in.^{4}, in order to approximate steel. The stiffness of axial springs was 1.60×10^4 lbf/in., approximating relatively soft boundary conditions. The beam model was meshed with 40 two node beam elements to have 117 free DOF. Then, two design variables for the optimization were chosen among the FE model parameters, i.e. the radius of curvature that can capture the imperfection of curvature of the beam and the axial spring stiffness that can identify the boundary conditions, based on an assumption that these two physical aspects are uncertain. Meanwhile, a target ROM of the beam model was generated from a reference FE model that has small differences in design parameters compared to the original beam model. In this example, the reference FE model was identical to the beam model except for the radius of curvature, $R = 100$ in. (i.e. 125% of the original beam model) and the axial boundary stiffness, $K_x = 2.00 \times 10^4$ lbf/in. (i.e. 125% of the original beam model).

In this example, using a single mode ROM composed of the first bending mode was sufficient to accurately update the design variables of the beam model. The linear and nonlinear coefficients of a single mode ROM of the initial beam model as well as those of the target ROM are presented in Table 1. The optimization took 13 iterations to minimize the gap between the coefficients of the two ROMs as shown in Figure 3. The converged curves of two design variables in Figure 4 also demonstrates that the tuning is accurately performed with relatively small number of iterations. The percentage errors of the updated design variables with respect to the reference values were below 0.0001% at 8th iteration.

It is important to mention that the design parameters should be properly chosen in order to effectively account for the nonlinear stiffness terms of the updating ROM. For example, the beam thickness captures only the linear behaviors of the beam model, so that the parameter could never contribute to updating the nonlinear terms. On the other hand, the radius of curvature of the beam and the axial spring stiffness were able to account for both linear and nonlinear stiffnesses, which resulted in an accurate tuning. Also, note that the gradient-based optimization used in this work provides only a local minimum. Thus, it is possible that the updated design variables may not be the true values, although it was not the case in the presented example. More complex examples that can further explore these issues will be discussed during the presentation.

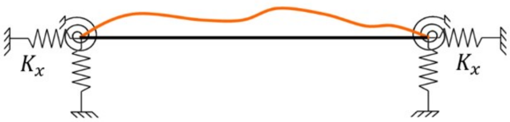


Figure 2: Schematic of a beam with uncertain radius of curvature R and axial stiffness terms K_x

Coefficients	Initial Value	Final Value	Target Value
ω_r	907.8337	833.9752	833.9752
$A_1(1,1)$	-1.3458×10^9	-1.2828×10^9	-1.2828×10^9
$B_1(1,1,1)$	6.1813×10^{11}	7.5551×10^{11}	7.5551×10^{11}

Table 1: Linear and nonlinear stiffness coefficients of the optimized ROM and the target ROM

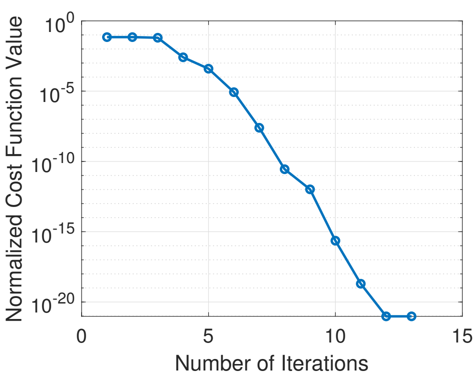


Figure 3: Convergence history of cost function value normalized by the target ROM coefficients

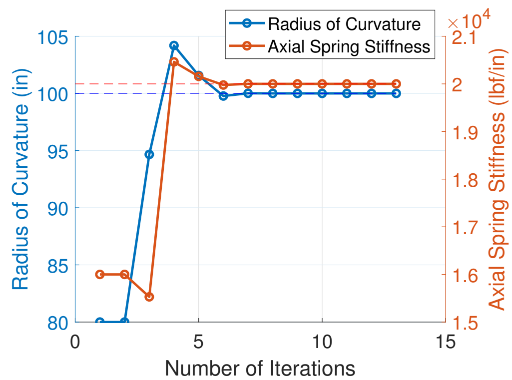


Figure 4: Convergence history of the design variables

Acknowledgments

This work was supported by the Air Force Office of Scientific Research, Award # FA9550-17-1-0009, under the Multi-Scale Structural Mechanics and Prognosis program managed by Dr. Jaimie Tiley. The authors would also like to thank Christopher I. Van Damme who contributed to our numerical study.

References

[1] Christopher I Van Damme. *Model correlation and updating of geometrically nonlinear structural models using nonlinear normal modes and the multi-harmonic balance method.* PhD dissertation, University of Wisconsin-Madison, 2019.

[2] Robert J Kuether, Brandon J Deaner, Joseph J Hollkamp, and Matthew S Allen. Evaluation of geometrically nonlinear reduced-order models with nonlinear normal modes. *AIAA Journal*, 53(11):3273–3285, 2015.

[3] Richard H Byrd, Jean Charles Gilbert, and Jorge Nocedal. A trust region method based on interior point techniques for nonlinear programming. *Mathematical programming*, 89(1):149–185, 2000.