Evaluation of Interface Reduction Methods for Craig Bampton Models

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1. Introduction

Large and complicated structures often consist of multiple components that can be readily decomposed into an assembly of relatively smaller subsystems. Component mode synthesis (CMS), also referred to as dynamic substructuring, is widely used in structural dynamics to assemble the reduced order models of these substructures and form the global system [1]. The Craig-Bampton (CB) approach [2] is one of the most popular CMS techniques. For a finite element model with a very fine and detailed mesh at the interface, the CB model is dominated by the number interface DOF. The scope of this research is to evaluate and compare a variety of methods developed to reduce the interface for traditional CB substructuring.

To the best of the authors' knowledge, the earliest work on interface reduction was by Craig and Chang in [3]. They suggested three separate methods to reduce the number of interface DOF in 1977, but their methods do not seem to have been adopted in industry. Later, Castanier et al. [4] rediscovered one of the three methods by applying a secondary eigenvalue analysis at the interface partition of the assembled CB models, resulting in the so-called system-level characteristic-constraint (SCC) modes. These SCC modes can be truncated much like the fixed-interface modes and used to further reduce the CMS model. Hong et al. [5] proposed an alternate interface reduction technique by applying the secondary eigenvalue analyses at the substructure level prior to the assembly step, resulting in a set of local-level characteristic-constraint (LCC) modes. In their approach, interface compatibility is guaranteed by joining all the LCC modes for each interface between different subsystems. An alternate approach was recently proposed by Kuether et al. in [6] where the LCC basis was used to reduce the interface DOF and used a weak compatibility approach to assemble the subcomponents. A generalized extension to the connection of multiple substructures can be done using the modal constraints developed in [7]. Another type of interface reduction was proposed by Lindberg et al. in [8] using a set of undeformed interface coupling modes. This technique constrains the interface to six rigid body translations and rotations and is valid for coupling between subcomponents with relatively different stiffnesses.

This extended abstract describes the research performed at the 2016 Nonlinear Mechanics and Dynamics (NOMAD) Research Institute hosted by Sandia National Laboratories. The objective of this project was to investigate the accuracy and applicability of a variety of existing interface reduction techniques. Section 2 of this extended abstract reviews the classical Craig-Bampton method and four interface reduction methods of interest, and introduces a novel hybrid interface reduction technique. Section 3 presents the finite element model used to compare the different techniques.

2. Interface Reduction Techniques

2.1. Craig Bampton Reduction

Each of the interface reduction methods is applied as a secondary reduction after a CB model order reduction has been performed. The CB representation uses a truncated set of fixed-interface modes as a basis for deformation in the interior of each substructure, and a set of constraint modes as the deformation basis of each substructure's interface(s). The fixed-interface modes of the jth substructure, Φ_j, are normal vibration modes obtained by holding the interface fixed. The constraint modes, Ψ_j, are obtained by applying a unit deflection to each interface DOF with all other interface DOF fixed, and computing the corresponding static displacement of the interior. Combining these mode sets gives a transformation from the full model DOF vector to the CB DOF vector,

$$\left\{\begin{matrix}\mathbf{u}_{i,j}\\ \mathbf{u}_{b,j}\end{matrix}\right\} = \begin{bmatrix}\Phi_j & \Psi_j\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\left\{\begin{matrix}\mathbf{q}_{i,j}\\ \mathbf{u}_{b,j}\end{matrix}\right\}. \tag{1}$$

The internal DOF are transformed from physical coordinates, $\mathbf{u}_{i,j}$, to a reduced set of modal coordinates, $\mathbf{q}_{i,j}$. The physical coordinates at the interface, $\mathbf{u}_{b,j}$, remain unchanged. Pre- and post-multiplying the full model mass and stiffness matrices for the jth substructure with the CB transformation yields substructure CB mass and stiffness matrices, $\mathbf{M}_j^{CB}$ and $\mathbf{K}_j^{CB}$. Since the interface DOF are unchanged, the reduced model substructures can be directly coupled together using the direct stiffness method to get the CB system mass and stiffness matrices, $\mathbf{M}^{CB}$ and $\mathbf{K}^{CB}$.

2.2. *System-level Interface Reduction*

Craig and Chang [3] and later Castanier et al. [4] presented a system-level technique for reducing the size of the CMS model's interface DOF. They obtained the characteristic-constraint modes by performing a secondary eigenvalue analysis on the interface partition of the assembled system,

$$\left(\mathbf{K}_{bb}^{CB} - \omega_{CC}^2\mathbf{M}_{bb}^{CB}\right)\phi^{CC} = \mathbf{0}. \tag{2}$$

The vector ϕ^{CC} is the SCC eigenvector of the interface partition of the assembled system and ω_{CC} is the corresponding eigenvalue. In general, a truncated set of these SCC modes are used to reduce the number of interface DOF of the CMS model. The advantage of this approach is that the basis is computed from the assembled system, so all interface motions are captured accurately and a highly efficient basis is obtained.

2.3. *Local-level Interface Reduction*

A useful feature of the CB representation is that reduced order models for each substructure are local, that is they do not depend on how the substructures are assembled. The SCC reduction performs the secondary modal analysis using the assembled stiffness and mass matrices. However, the advantages of local substructure models are lost since the basis requires knowledge of the adjacent structure.

Three variations of local-level interface reduction techniques are described in this subsection where the reduction is accomplished before the assembly step. For the j^{th} CB model, the secondary eigenvalue analyses on the interface DOFs (denoted here as subscript b) can be written as,

$$\left(\mathbf{K}_{bb,j}^{CB} - \omega_{LCC,j}^2\mathbf{M}_{bb,j}^{CB}\right)\phi_j^{LCC} = \mathbf{0}, \tag{3}$$

where ϕ_j^{LCC} is an LCC mode of j^{th} substructure and $\omega_{LCC,j}$ is the corresponding eigenvalue. A few variations exist the LCC modal basis, mainly differing by how compatibility is satisfied during the assembly step.

2.3.1. *Exact-Compatibility [5]*

In the Exact Compatibility approach of Hong et al. [5], the LCC modes for each substructure interface are simply concatenated. This augmented set of CC modes is used as the reduction basis for the interface of every connected substructure. This guarantees that the interface dynamics of each connected substructure are well described, at the cost of increasing the size of the basis. Using the same modes for each connecting substructure also guarantees that interface modes are exactly compatible.

Due to the concatenation of multiple mode sets, there is no guarantee that the augmented CC mode sets are linearly independent. (Indeed, in some cases the interface modes may be nearly identical so one has an interface set with multiple copies of each interface mode.) To guard against ill-conditioning, the augmented set of modes should be orthonormalized, and in practice the set is also reduced to eliminate redundant shapes. In Hong's work and here this is done by performing a singular value decomposition and keeping only the left singular vectors corresponding to singular values larger that 0.01% of the maximum singular value.

2.3.2. *Weak-Compatibility [7]*

Rather than augmenting the interface mode sets so that compatibility can be enforced directly, the weak compatibility method simply uses the local interface modes directly to reduce each substructure interface. The resulting reduced subcomponents can no longer be coupled directly because in general the interface mode shapes will not exactly match the interface mode shapes from the adjacent substructure(s). In fact, the interface mode sets typically will not even span the same space. Thus, compatibility is weakly enforced by projecting the interface deformations from each substructure onto a common set of generalized coordinates that minimize compatibility error between substructures.

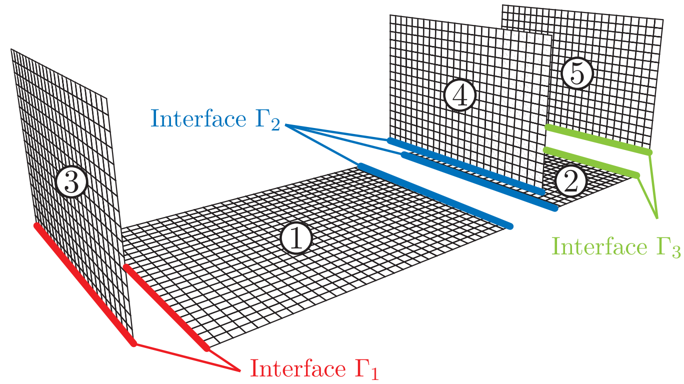

Figure 1: Exploded view of the W-Bracket system consisting of five plate substructures. The three interface sets are highlighted and labeled.

2.3.3. Weak-Compatibility Uncoupled [6]

The weak compatibility technique described in the previous section results in a single coupled set of interface DOF for the system. This means that the different interface sets cannot move independently. Another approach to weak compatibility interface reduction is to decouple the interface modes before enforcing weak compatibility so that the interface sets *can* move independently. This increases the number of interface DOFs in the system but can make the interface reduction easier to apply in some cases.

2.3.4. Undeformed Coupling Interface Modes [8]

Another local-level reduction technique is the undeformed interface coupling modes proposed by Lindberg et al. in [8]. This method was developed in order to reduce the number of interface DOF required to connect stiff and soft substructures by creating a virtual node and constraining the interface to displace or rotate with respect to this node. The CB subcomponent models use the fixed-inteface modes to capture the interior DOF. However, instead of the classical static constraint modes, the boundary DOF are allowed $\mathbf{u}_{b,j}$ to move in six shapes defined by three orthogonal translations and three rotations around a virtual node.

2.4. Hybrid Reduction

Although the system-level interface reduction generally produces accurate results by properly considering the coupling between all the substructures, the secondary eigenvalue analysis must be applied after all the subcomponents have been assembled. It loses some of the advantage of substructuring where the reduction can be applied locally. The local-level interface reduction has the advantage of being applied at the substructure level, however, loses accuracy since the coupling between different substructures is totally neglected during the secondary modal analysis. A hybrid interface reduction technique is proposed here where the secondary eigenvalue analysis is performed at a local interface shared by two or more substructures, rather than the whole set of interfaces as done with the system-level approach. By doing so, the coupling between neighboring substructures, which is of great significance, can be taken into consideration and one only needs the mass and stiffness local to each interface.

3. Numerical Example

Each of the interface reduction techniques have been applied to the numerical example shown in Fig. 1. This model was originally studied in [5] for local-level interface reduction. The structure has a number of salient features that make it an excellent benchmark for interface reduction techniques. It contains more than one interface, at least one interface connects three or more substructures, and the bending directions of the substructures are perpendicular to one another. In the IMAC presentation the accuracy and efficiency of the methods will be compared and some general conclusions can be drawn from the results.

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